Преглед НЦД 33 (2018), 25–34

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DIGITIZED UNIVERSITY TEXTBOOK OF PROF. ZAHARIJE BRKIĆ IN VIRTUAL LIBRARY OF FACULTY OF MATHEMATICS

Abstract. In the Virtual Library of the Faculty of Mathematics there are three digitized publications by professor Zaharije Brkić. These publications include his doctoral dissertation, which represents the first dissertation in astronomy that had been defended on Belgrade university. Also, two university textbooks written with prof. Branislav Ševarlić are digitized. In this paper, we give detailed analysis of these works along with explanations of their importance.

1. Introduction

Figure 1. Professor Zaharije Brkić photographed during the observations of total solar eclipse on Hvar (Croatia) in 1961 [3].

For the purpose of keeping and preserving a large number of rare books, textbooks, dissertations and papers, Virtual Library was built as a part of the Digitization project of the Faculty of Mathematics in Belgrade and the Mathematical Institute of the Serbian Academy of Sciences and Arts [1, 2]. Virtual Library is freely available and open via Internet link http://elibrary.matf.bg.ac.rs for general public. This project will allow future generations to better understand works of our prominent scientists, as well as to save past works from oblivion.

In this paper, we present digitized university textbooks written by Zaharije Brkić, who was an astronomer and respected professor at the University of Belgrade. Paper is organized as follows. In the next section we give a short biography of professor Zaharije Brkić. Third, fourth and fifth sections contain detailed analysis of two digitized textbooks and his dissertation.

2. Biography of professor Zaharije Brkić (1910-1979)

Zaharije Brkić (Figure 1) was born in Poljna (central Serbia, near municipality of Trstenik) at 1910. As a young pupil he showed great interest and affinity towards mathematics and physics. After he had finished grammar school in Čačak he started studying at the Faculty of Philosophy in Belgrade, as part of a group for theoretical mathematics. He graduated at 1936 and two years afterwards he begun to work at the Astronomical observatory Belgrade. At observatory he was first an assistant but soon was promoted to researcher position. Due to his extensive knowledge of astronomical instruments, their usage and application for position determination he started working as an assistant professor on the Faculty of Civil Engineering at 1955. Two years after, on 1957. he was elected as a professor and he remained on that positions until he moved to Faculty of Natural sciences [3, 4].

Zaharije Brkić defended his doctoral dissertation in astronomy at Belgrade university on 26th September 1958. This was the first dissertation to have been defended at Belgrade university in astronomical sciences. His mentor, Vojislav Mišković, was the first person that had obtained doctorate in astronomy. On board for his defense were Tatomir Anđelić, full professor, Vojislav Mišković, full professor and academic, and Konstantin Voronjec as a corespondent member. One year after he got his doctoral degree he moved to department for mechanics and astronomy on Faculty of Natural sciences where he had been working until his retirement on July 1978. During his work at faculty, Zaharije Brkić was on board for multiple doctoral defenses: Branislav Ševarlić (1960), Vasilije Oskanjan (1961), Đorđe Teleki (1964) and Dragutin Đurović (1974) [3, 4].

Zaharije Brkić was one of the founders of the services for (1) time and changing of geographic latitude and (2) for longitude change and movement of Earth poles at the Astronomical observatory. Also, he published 34 scientific papers and 27 professional papers. Besides that, he is coauthor of two university textbooks: Geodetic astronomy and General astronomy. We should mentioned that General astronomy has two editions and it is still in use today. Both textbooks are digitized and available at Virtual Library. More detail about textbooks are given in the following sections.

Zaharije Brkić was married to Marica Marković with who he had two children: daughter Ružica and son Slobodan. Professor Zaharije Brkić died on 24th April 1979 after fighting a difficult desease. Shortly before his death, he was given medal for his scientific achievements and his influence on education for numerous generations of astronomers.

3. Doctoral Dissertation

As it was previous mentioned, doctoral dissetation written by Zaharije Brkić, was the first dissertation in astronomy defended at the University of Belgrade. Its title is "Analiza sistematskih gresaka pasažnog instrumenta i drugih sistematskih uticaja na određivanje vremena" (The analysis of systematic errors of passage instrument and other systematic influences on time determination). His dissertation is publicly available on link: http://elibrary.matf.bg.ac.rs/handle/123456789/22 [7].

Dissertation which was digitized is available in the library of the Faculty of Mathematics where it is classified as museum copy under the catalogue number "Dokt. 4/1". Unfortunately, this copy is damaged and some pages are barely visible (Figure 2).

Dissertation has 120 pages and its content is divided in two parts. First part is a description of chosen problem and historical remarks. Second part is where Zaharije Brkić presents his own scientific work. Whole dissertation is written using typewriter and sufficient copies are obtained using indigo paper. Graphics were multiplied using photosensitive paper which is a reason for gradually degrading of their quality (Figure 2). Historic and scientific significance of this dissertation makes this digitization and its preservation.

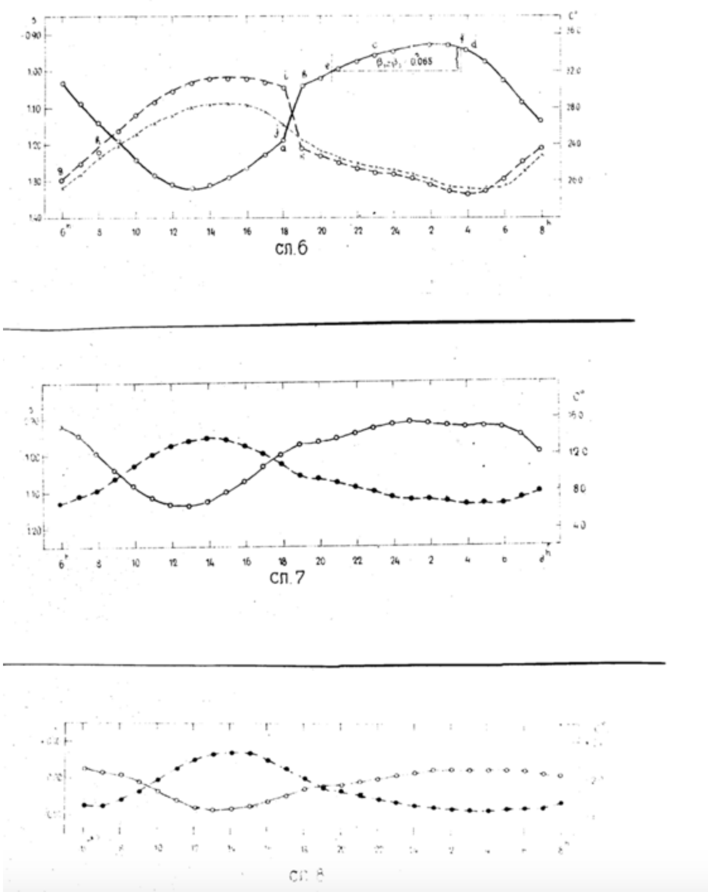


Figure 2. One page at the digitized version of doctoral dissertation of prof. Brkić. On this page, damages are clearly visible.

Authors of this paper have found that Miodrag Dačić has one copy of Zaharije Brkić's doctoral dissertation which is not damaged, and its digitization is on the way, and soon it will be available at the same link as already digitized version.

3. Geodetic astronomy

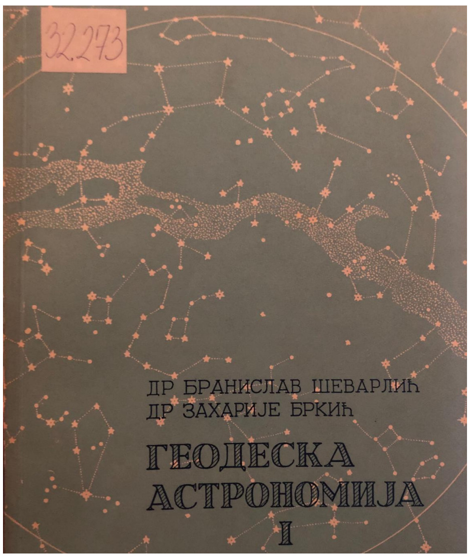


Figure 3. Front page of the textbook Geodetic astronomy.

"Gedeska astronomija" (Geodetic astronomy) was published in 1963. and it represents the first astronomy textbook for the students of the Geodesy department of the Faculty of Civil Engineering. Textbook has 266 pages, with 63 solved problems, 92 illustrations and 33 tables. Solved problems are very useful for students as they help them to overcome and easily accept new terms. In the following paragraphs we are showing some selected parts of this textbook.

On Figure 4 (pages 122 and 123 from textbook) we can see one solved problem (problem no. 49). This problem shows a method for calculation of the time of the Sun's setting and rising, but accounting for the atmospheric refraction. In this problem, duration of the day is calculated for Belgrade and Zagreb, and in order to achieve a larger precision, problem is done using two approximations. At that time calculators and computers were not in use, hence extensive calculations had to be done using logarithmic tables or mechanical calculators.

Here, we should mention that completely solved problems with explanations can be found in "Zbirka urađenih zadataka iz Opšte astronomije" (Solved problems in general astronomy) by Vojislav Mišković [5, 6]. The first part of this problem book (published in 1957) and second part (as manuscript) is freely available at Virtual Library (http://elibrary.matf.bg.ac.rs/handle/123456789/650). This book is still the only one problem book in general astronomy.

Discovery and studying of irregularities of the Earth rotation, especially secular slowing down of its rotation led to new astronomical time: ephemeris time (ET). On Figure 5 two pages from the textbook are shown where authors explained ET using graphical and analytical way. Also, the relations between ephemeris time, universal time and sidereal time are given. From the expression 336 (L_E = L + $1.00273\ \Delta t$) for geographic longitude of ephemeris meridian, we see that the ephemeris meridian is constantly moving away from the Greenwich meridian. This is the consequence of secular slowing down of the Earth rotation period. Determination of a correct longitude is greatly important for the geodesy.

Before determination of the irregularities of Earth rotation, the movements of the Earth poles had been discovered. Therefore, besides changes in longitude we have constant changes in latitude for every place on the Earth. In the Geodetic astronomy authors are deriving the equation of Kostinsky for determination of cartesian coordinates of Earth pole form observational data. It is important to note that the first mathematical model of Earth, and movement of its poles was discovered by our scientist Milutin Milanković. Milutin Milanković gave theoretical derivation of this Earth model in his monumental work "Canon of insolation and ice age problem".

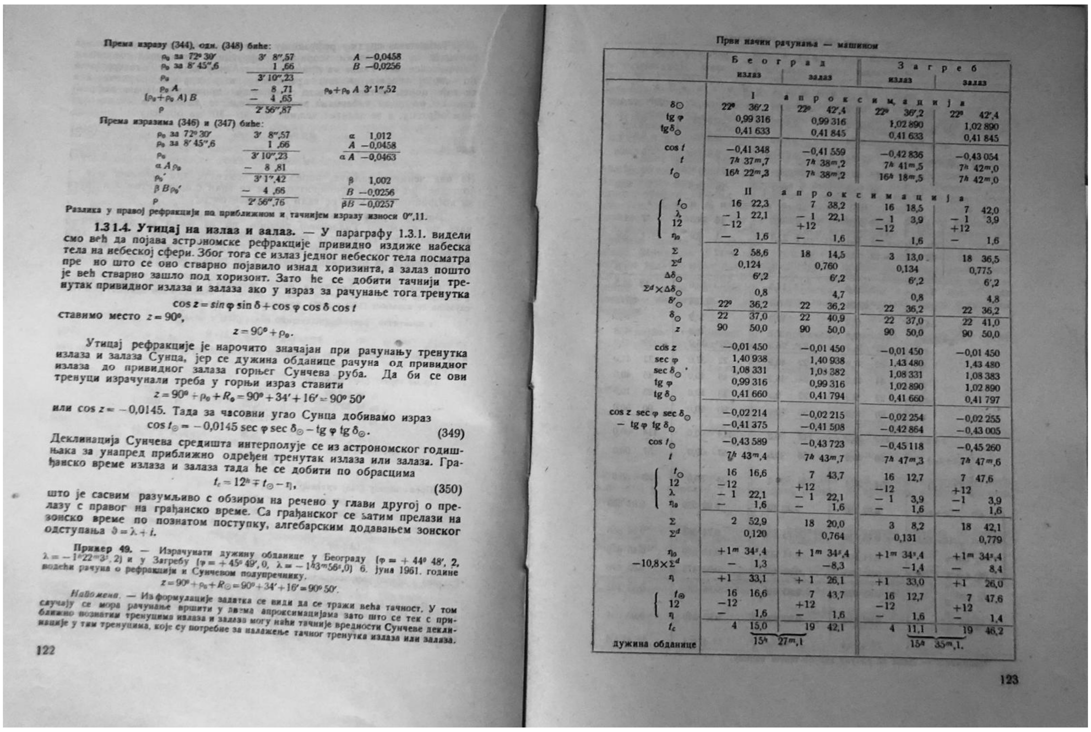


Figure 4. Scan of the pages 122 and 123 from Geodetic astronomy.

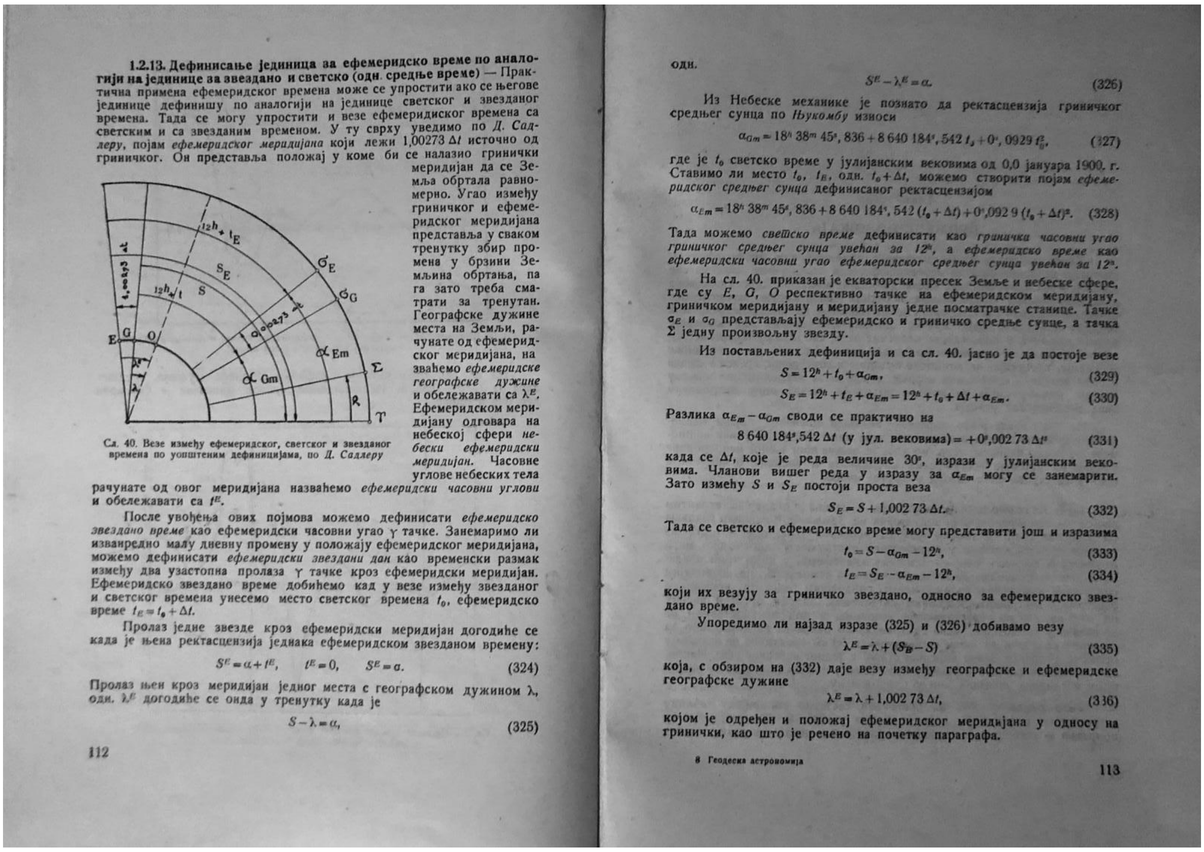


Figure 5. Scan of the pages 112 and 113 from Geodetic astronomy.

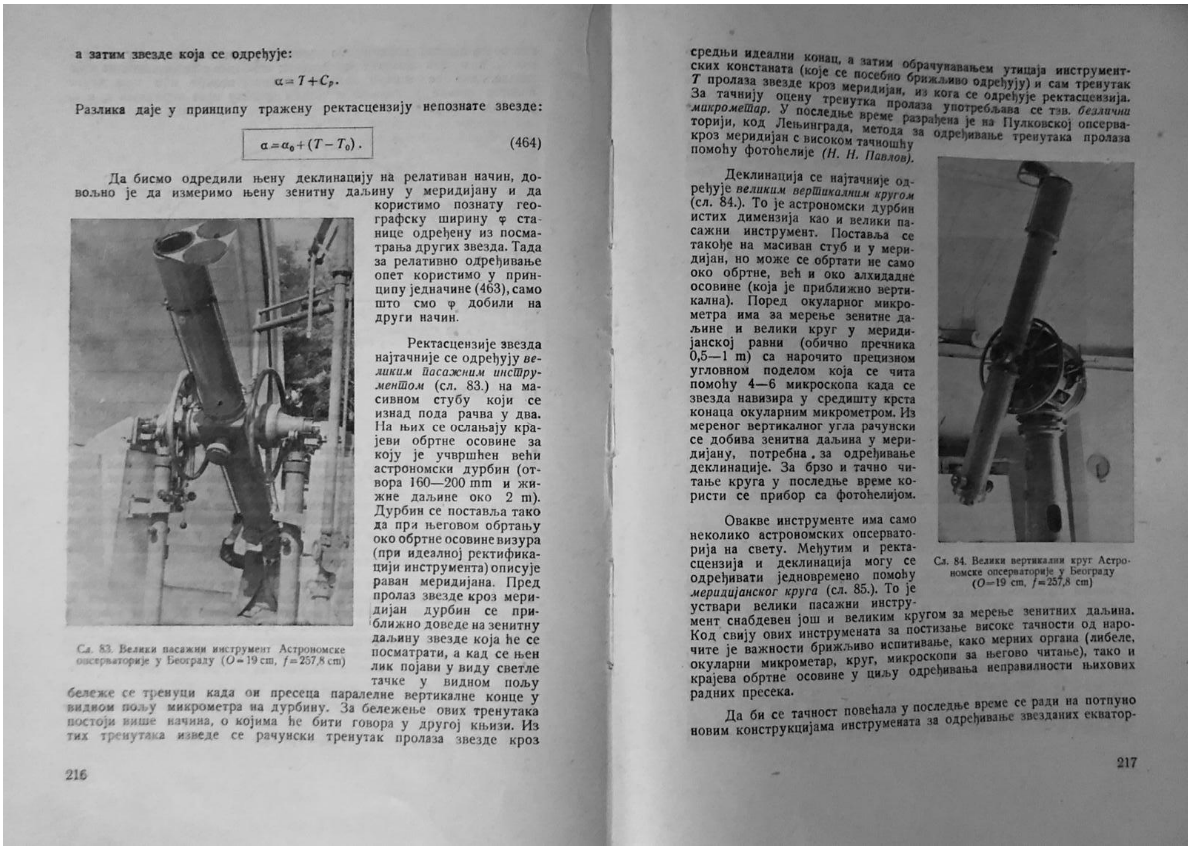


Figure 6. Scan of the pages 216 and 217 from Geodetic astronomy

On pages 216 and 217 (Figure 6), we can see pictures (photos) of two important astronomical instruments: large passage instrument and large vertical circle from the Astronomical observatory Belgrade. Large passage instrument gives the most precise determination of right accession, and large vertical circle is used for declination determination. Using these two instruments observational catalogues were made. Unfortunately, due to light pollution these instruments are no longer in use at the observatory.

We should mention that for a relative coordinate determination of the equatorial coordinates of stars, meridian circle is used. Meridian circle can estimate both coordinates at the same time. Unhappily, this instrument was destroyed in fire during the NATO bombing of Serbia in 1999.

4. General Astronomy

"Opšta astronomija" (General astronomy) is one of the most significant textbook for students of astronomy. This book is still in extensive use for the first year students as material for the courses General astronomy 1 and General astronomy 2. The first edition of this book was published in 1971., and the second one ten years later (

Figure 7). General astronomy treats the basics of the classical astronomy disciplines such as: spherical, practical, positional and theoretical astronomy, celestial mechanics etc. Here, we should highlight that every chapter in this book starts with historical background and also is followed by graphical representation of the problem.

Figure 7. Front pages of the university textbook General astronomy. Left one is published in 1971., while the right one is the second edition published in 1981.

Second edition of this book is extended with newer data and solved problems. Also, the profit from the sale of this edition went to Fond of prof. Zaharije Brkić whose purpose is to give a reword for the best graduated astronomy students.

On pages 34 and 35 (Figure 8) it is shown how to use equations of spherical trigonometry in order to do transformations between spherical coordinate systems that are used in astronomy. There we can see one image (image number 12 in textbook) of the celestial sphere and spherical triangle that is used to derive the relations between equatorial and ecliptic coordinates. At chapter 70 (Figure 9) determination of true solar time with sundial is shown. With spherical trigonometry it is possible to obtain the relation between the shadow of gnomon and Sun's hour angle. At Belgrade there are a few sundials: in front of a mint of money at Topčider, at the entrance of the Astronomical observatory Belgrade, in front of Serbian clinical centre, in the centre of Zemun and at the representative office of the Hilandar monastery.

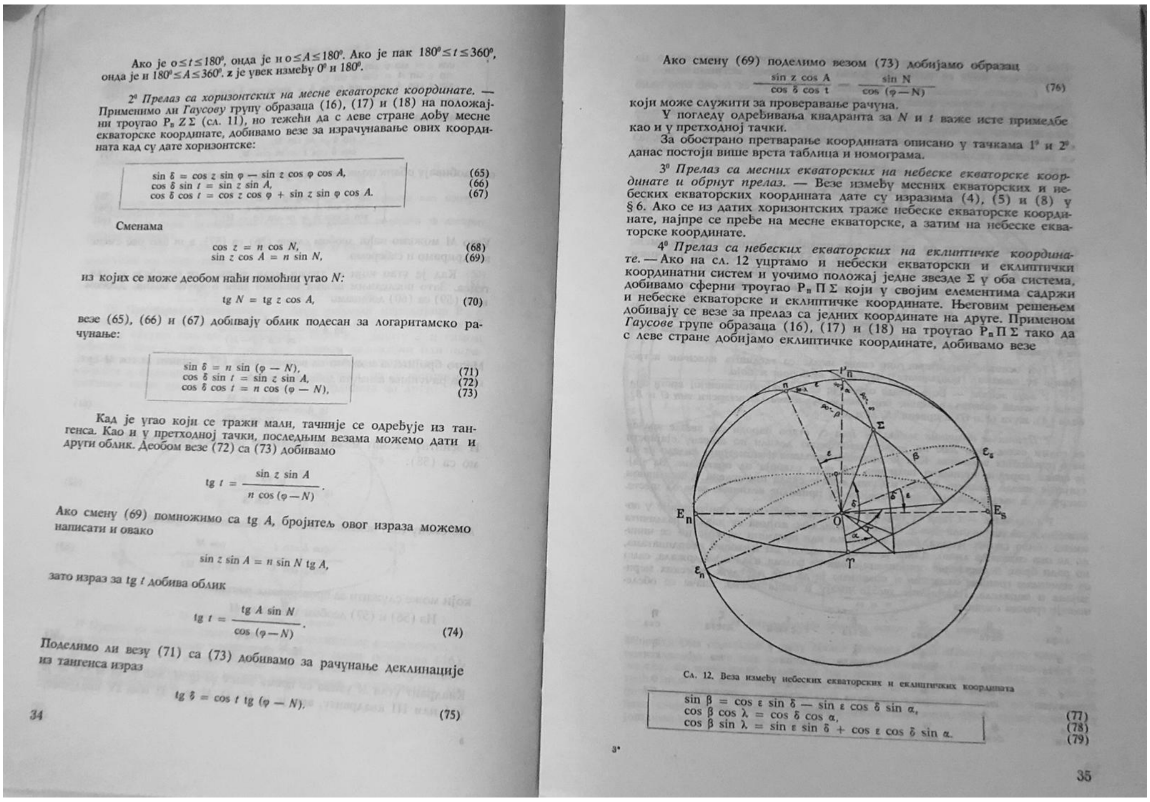
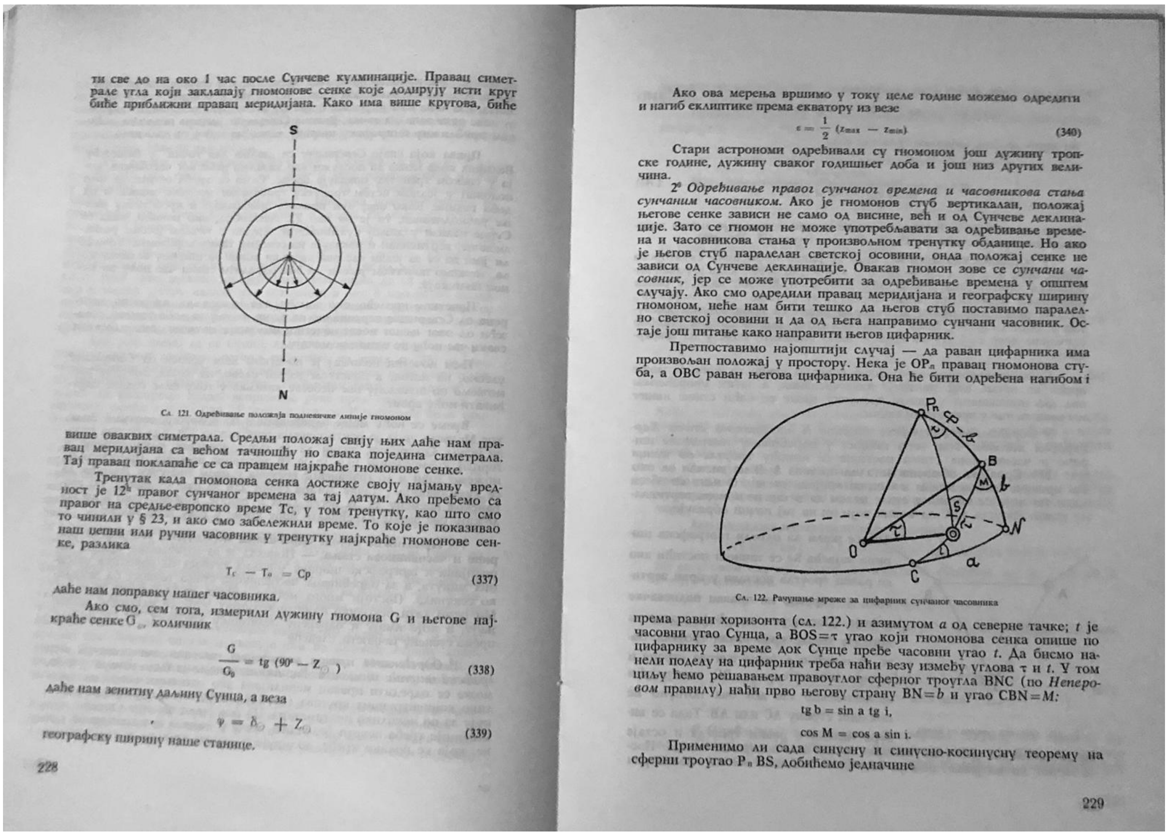


Figure 8. Pages 34 and 25 from the textbook General astronomy.



ти све до на око 1 час после Сунчеве кулминације. Правац симетрале угла који заклапају гномонове сенке које додирују исти круг биће приближни правац меридијана. Како има више кругова, биће

Сл. 121. Одређивање положаја подневичке линије гномоном

више оваквих симетрала. Средњи положај свију њих даће нам правац меридијана са већом тачношћу но свака поједина симетрала. Тај правац поклапаће се са правцем најкраће гномонове сенке.

Тренутак када гномонова сенка достиже своју најмању вредност је 12^h правог сунчаног времена за тај датум. Ако пређемо са правог на средње-европско време Tc, у том тренутку, као што смо то чинили у § 23, и ако смо забележили време. То које је показивао наш џепни или ручни часовник у тренутку најкраће гномонове сенке, разлика

$$T_c - T_0 = Cp \tag{337}$$

даће нам поправку нашег часовника.

Ако смо, сем тога, измерили дужину гномона G и његове најкраће сенке G_0, количник

$$\frac{G}{G_0} = \text{tg}\,(90° - Z_\odot) \tag{338}$$

даће нам зенитну даљину Сунца, а веза

$$\varphi = \delta_\odot + Z_\odot \tag{339}$$

географску ширину наше станице.

228

Ако ова мерења вршимо у току целе године можемо одредити и нагиб еклиптике према екватору из везе

$$\varepsilon = \frac{1}{2}\,(z_{max} - z_{min}) \tag{340}$$

Стари астрономи одређивали су гномоном још дужину тропске године, дужину сваког годишњег доба и још низ других величина.

2° *Одређивање правог сунчаног времена и часовникова стања сунчаним часовником.* Ако је гномонов стуб вертикалан, положај његове сенке зависи не само од висине, већ и од Сунчеве деклинације. Зато се гномон не може употребљавати за одређивање времена и часовникова стања у произвољном тренутку обданице. Но ако је његов стуб паралелан светској осовини, онда положај сенке не зависи од Сунчеве деклинације. Овакав гномон зове се *сунчани часовник*, јер се може употребити за одређивање времена у општем случају. Ако смо одредили правац меридијана и географску ширину гномоном, неће нам бити тешко да његов стуб поставимо паралелно светској осовини и да од њега направимо сунчани часовник. Остаје још питање како направити његов цифарник.

Претпоставимо најопштији случај — да раван цифарника има произвољан положај у простору. Нека је OP_n правац гномонова стуба, а OBC раван његова цифарника. Она ће бити одређена нагибом i

Сл. 122. Рачунање мреже за цифарник сунчаног часовника

према равни хоризонта (сл. 122.) и азимутом *a* од северне тачке; *t* је часовни угао Сунца, а BOS=τ угао који гномонова сенка опише по цифарнику за време док Сунце пређе часовни угао *t*. Да бисмо нанели поделу на цифарник треба наћи везу између углова τ и *t*. У том циљу ћемо решавањем правоуглог сферног троугла BNC (по *Неперовом* правилу) наћи прво његову страну BN=*b* и угао CBN=*M*:

$$\text{tg}\, b = \sin a\ \text{tg}\, i,$$

$$\cos M = \cos a \sin i.$$

Применимо ли сада синусну и синусно-косинусну теорему на сферни троугао P_n BS, добићемо једначине

229

Figure 9. Pages 228 and 229 from the textbook General astronomy.

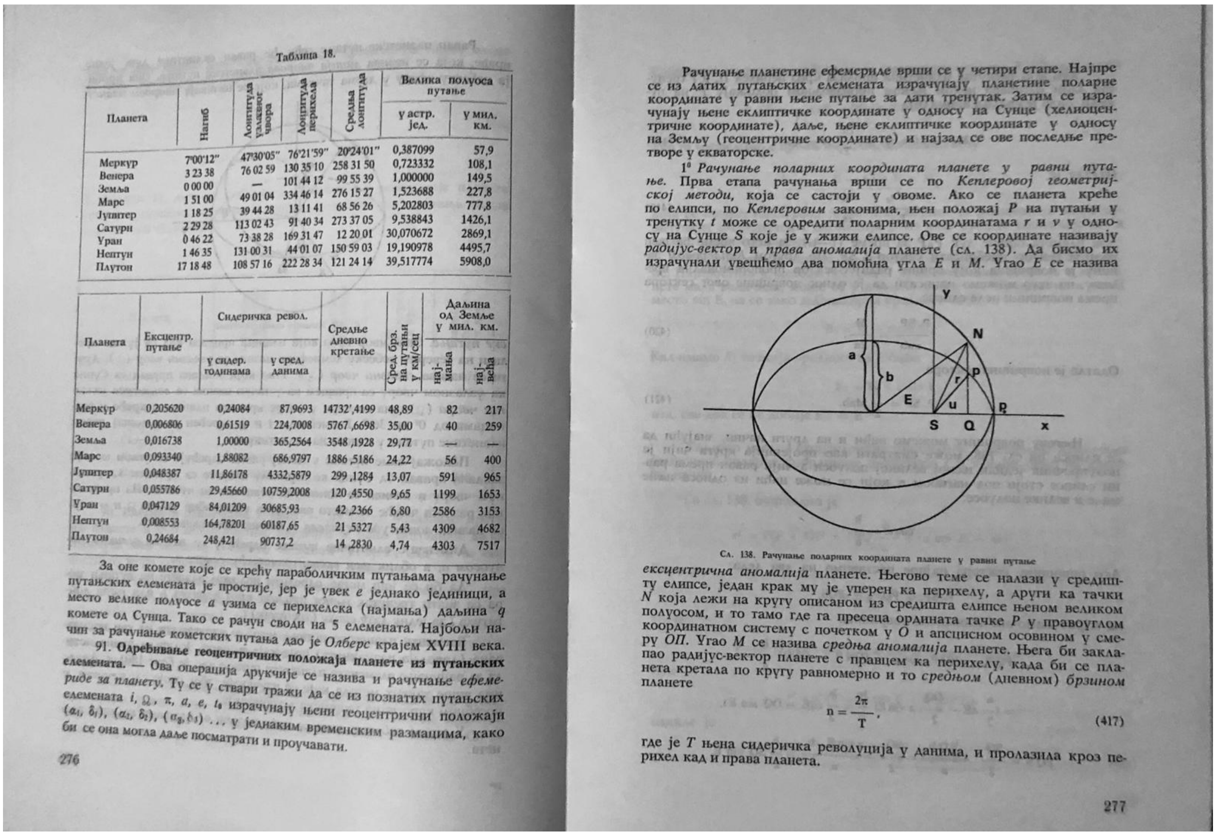


Таблица 18.

Планета	Нагиб	Лонгитуда узлазног чвора	Лонгитуда перихела	Средња лонгитуда	Велика полуоса путање у астр. јед.	Велика полуоса путање у мил. км.
Меркур	7°00'12"	47°30'05"	76°21'59"	20°24'01"	0,387099	57,9
Венера	3 23 38	76 02 59	130 35 10	258 31 50	0,723332	108,1
Земља	0 00 00	—	101 44 12	99 55 39	1,000000	149,5
Марс	1 51 00	49 01 04	334 46 14	276 15 27	1,523688	227,8
Јупитер	1 18 25	39 44 28	13 11 41	68 56 26	5,202803	777,8
Сатурн	2 29 28	113 02 43	91 40 34	273 37 05	9,538843	1426,1
Уран	0 46 22	73 38 28	169 31 47	12 20 01	30,070672	2869,1
Нептун	1 46 35	131 00 31	44 01 07	150 59 03	19,190978	4495,7
Плутон	17 18 48	108 57 16	222 28 34	121 24 14	39,517774	5908,0

Планета	Ексцентр. путање	Сидеричкаревол. у сидер. годинама	Сидеричкаревол. у сред. данима	Средње дневно кретање	Сред. брз. на путањи у км/сец	Даљина од Земље у мил. км. нај-мања	Даљина од Земље у мил. км. нај-већа
Меркур	0,205620	0,24084	87,9693	14732',4199	48,89	82	217
Венера	0,006806	0,61519	224,7008	5767 ,6698	35,00	40	259
Земља	0,016738	1,00000	365,2564	3548 ,1928	29,77	—	—
Марс	0,093340	1,88082	686,9797	1886 ,5186	24,22	56	400
Јупитер	0,048387	11,86178	4332,5879	299 ,1284	13,07	591	965
Сатурн	0,055786	29,45660	10759,2008	120 ,4550	9,65	1199	1653
Уран	0,047129	84,01209	30685,93	42 ,2366	6,80	2586	3153
Нептун	0,008553	164,78201	60187,65	21 ,5327	5,43	4309	4682
Плутон	0,24684	248,421	90737,2	14 ,2830	4,74	4303	7517

За оне комете које се крећу параболичким путањама рачунање путањских елемената је простије, јер је увек *e* једнако јединици, а место велике полуосе *a* узима се перихелска (најмања) даљина *q* комете од Сунца. Тако се рачун своди на 5 елемената. Најбољи начин за рачунање кометских путања дао је *Олберс* крајем XVIII века.

91. **Одређивање геоцентричних положаја планете из путањских елемената.** — Ова операција друкчије се назива и рачунање *ефемериде за планету*. Ту се у ствари тражи да се из познатих путањских елемената *i*, ☊, π, *a*, *e*, t_0 израчунају њени геоцентрични положаји (α_1, δ_1), (α_2, δ_2), (α_3, δ_3) ... у једнаким временским размацима, како би се она могла даље посматрати и проучавати.

276

Рачунање планетине ефемериде врши се у четири етапе. Најпре се из датих путањских елемената израчунају планетине поларне координате у равни њене путање за дати тренутак. Затим се израчунају њене еклиптичке координате у односу на Сунце (хелиоцентричне координате), даље, њене еклиптичке координате у односу на Земљу (геоцентричне координате) и најзад се ове последње претворе у екваторске.

1° *Рачунање поларних координата планете у равни путање.* Прва етапа рачунања врши се по *Кеплеровој геометријској методи*, која се састоји у овоме. Ако се планета креће по елипси, по *Кеплеровим* законима, њен положај *P* на путањи у тренутку *t* може се одредити поларним координатама *r* и *v* у односу на Сунце *S* које је у жижи елипсе. Ове се координате називају *радијус-вектор* и *права аномалија* планете (сл. 138). Да бисмо их израчунали увешћемо два помоћна угла *E* и *M*. Угао *E* се назива

Сл. 138. Рачунање поларних координата планете у равни путање

ексцентрична аномалија планете. Његово теме се налази у средишту елипсе, један крак му је уперен ка перихелу, а други ка тачки *N* која лежи на кругу описаном из средишта елипсе њеном великом полуосом, и то тамо где га пресеца ордината тачке *P* у правоуглом координатном систему с почетком у *O* и апсцисном осовином у смеру *OП*. Угао *M* се назива *средња аномалија* планете. Њега би заклапао радијус-вектор планете с правцем ка перихелу, када би се планета кретала по кругу равномерно и то *средњом* (дневном) *брзином* планете

$$n = \frac{2\pi}{T}, \tag{417}$$

где је *T* њена сидеричка револуција у данима, и пролазила кроз перихел кад и права планета.

277

Figure 10. Pages 276 and 277 from the textbook General astronomy.

Figure 10 shows another one classical term – Kepler's equation. There we can see that for the calculations of planet's polar coordinates we need Kepler's equation. Kepler's equation: $E - e \sin (E) = M$ is transcendental equation. Until the computer era solving of this equation was too hard.

Acknowledgment

This work was supported by the Serbian Ministry of Science, grant number III 44006.

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